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B.Sc - part-II(H). paper-III, Date-.18-05-2020'
Real Analysis. Topic- Conditional Convergence.

Q.1. Define Conditional Convergence with examples.

Soln. Conditional Convergence: →

If Σu_n is convergent and $\Sigma |u_n|$ divergent, then, Σu_n is said to be conditionally convergent. For examples:

Let us consider the series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\cdots \text{to } \infty$$

It is an alternating series.

Also, $|u_{n+1}| < |u_n|$ for all n and $u_n \to 0$ as $n \to \infty$.

Hence, by Leibnitz's test, $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ to ∞ is convergent

Also, $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$ to ∞ is divergent.

Hence, $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$ to ∞ is conditionally convergent.

Again, Let us consider the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ to ∞.

It is an alternating series.

Here, $|u_n| = \frac{1}{2n-1}$ and $|u_{n+1}| = \frac{1}{2n+1}$.

Since, $\frac{1}{2n+1} < \frac{1}{2n-1}$, therefore, $|u_{n+1}| < |u_n|$, for all n.

Also, $u_n \to 0$ as $n \to \infty$.

Hence, by Leibnitz's test the series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ to ∞ is convergent.

Also, $1+\frac{1}{3}+\frac{1}{5}+\frac{1}{7}+\cdots$ to ∞ is divergent.

Thus, $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$ to ∞ is conditionally convergent.

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Q. 2. Prove that the series $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\cdots$ to ∞ is ~~convergent~~ conditionally convergent.

Proof: Here, we see that $|u_n| = \frac{1}{\sqrt{n}}$ and $|u_{n+1}| = \frac{1}{\sqrt{n+1}}$

$\therefore |u_{n+1}| < |u_n|, \forall n.$

Also, $\lim_{n\to\infty} u_n = 0.$

The given series is an alternating series.
Hence, by Leibnitz's test, the given series $\sum u_n$ is convergent.
Again, $\sum |u_n| = 1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{4}}+\cdots$ to ∞, which is divergent.

Since, $\sum u_n$ converges but $\sum |u_n|$ diverges, therefore, the given series i.e. $\sum u_n$ is conditionally convergent.

Q.3. Discuss the convergence of the series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}$$

Soln: The given series $-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}-\frac{x^6}{6}+\cdots$ to ∞.

Here, there are two cases arise.

Case-I. If $0 < x \le 1$. $\lim_{n\to\infty} u_n = \lim_{n\to\infty} \frac{x^{n+1}}{n+1} = 0$ and the terms of the series go on decreasing.

We find that the series is alternating with terms decreasing and ultimately tending to zero.

Hence, by Leibnitz's rule, the given series is convergent if $0 < x \le 1$.

Case II. If $x > 1$, then $\frac{u_{n+1}}{u_n} = (-1)^{n+1}\frac{x^{n+2}}{n+2} \cdot \frac{n+1}{(-1)^n x^{n+1}}$

or, $\frac{u_{n+1}}{u_n} = -\frac{n+1}{n+2}x = \frac{1+\frac{1}{n}}{1+\frac{2}{n}}x$ or, $\lim_{n\to\infty}\frac{u_{n+1}}{u_n} = x > 1.$

So, the terms increase.
Clearly, the given series is alternating one.
Hence, it oscillates.

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